Physics
Ms. Brunsting

Calculations with Vectors

The Dot Product

When you add and subtract vectors, your answer is another vector. But when you calculate the dot product of two vectors, your answer is a scalar. It has no direction, just magnitude.

To calculate a dot product, multiply the x-components of both vectors, multiply the y-components of both vectors, then add these two numbers together.

$$\mathbf{A} \bullet \mathbf{B} = A_x \cdot B_x + A_y \cdot B_y$$

Dot products are used to calculate **work**. Work occurs when you use force to move an object a certain distance. If the force is applied at an angle, only part of the force is used to move the object. The part of the force that is useful is the part that is parallel to the direction of motion.

$$\textbf{Work} = \mathbf{F} \bullet \mathbf{d}$$

The Cross Product

A cross product is a way to multiply two vectors. The answer you get will be another vector. To calculate a cross product, multiply the x-component of the first vector by the y-component of the second. Then multiply the y-component of the first vector times the x-component of the second. Subtract the numbers to find the cross product.

$$\mathbf{A} \times \mathbf{B} = A_x \cdot B_y - A_y \cdot B_x$$

The cross product is used to calculate **torque**. Torque is a rotational force, like when you are twisting a bolt with a wrench.

To find the direction of the cross product vector, you need to use the Right Hand Rule. Take your right hand and point your fingers in the direction of **A**. Now curl your fingers toward **B**. Your thumb will point in the direction of the cross product vector. If your thumb points out of the page, the vector is positive. If your thumb points into the page, the vector is negative.

$$\textbf{Torque} = \mathbf{F} \times \mathbf{r}$$

Practice.

Find the dot product and cross product of each pair of vectors.

1. $\mathbf{A} = 6.1\,\mathbf{i} + 2.8\,\mathbf{j}$
   $\mathbf{B} = 3.9\,\mathbf{i} - 5.5\,\mathbf{j}$

2. $\mathbf{C} = 20.4\,\mathbf{i} - 16.9\,\mathbf{j}$
   $\mathbf{D} = 11.8\,\mathbf{i} + 36.3\,\mathbf{j}$

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Vector Homework

Convert each coordinate to unit vector notation.

1. (3, -2)

2. (7.5, 9.11)

3. (-4, 0.86)

Convert each magnitude and direction to unit vector notation.

4. Magnitude = 5, direction = 65°

5. Magnitude = 12.9, direction = 141°

Add or subtract the vectors.

6. $(2\,\hat{\imath} + 7\,\hat{\jmath}) + (8\,\hat{\imath} - 3\,\hat{\jmath})$

7. $(-13.9\,\hat{\imath} + 8.42\,\hat{\jmath}) - (5.06\,\hat{\imath} - 4.52\,\hat{\jmath})$

Find the product of the vectors.

8. $(12\,\hat{\imath} + 4\,\hat{\jmath}) \bullet (7\,\hat{\imath} + 2\,\hat{\jmath})$

9. $(2\,\hat{\imath} - 9\,\hat{\jmath}) \bullet (8.1\,\hat{\imath} + 3.4\,\hat{\jmath})$

10. $(5\,\hat{\imath} - 8\,\hat{\jmath}) \times (3\,\hat{\imath} + 6\,\hat{\jmath})$

11. $(6.38\,\hat{\imath} + 11.8\,\hat{\jmath}) \times (10.7\,\hat{\imath} + 2.26\,\hat{\jmath})$

12. A machine exerts a force of $62\,\hat{\imath} + 119\,\hat{\jmath}$ Newtons on a box. The box moves $81\,\hat{\imath} - 37\,\hat{\jmath}$ meters. Calculate the amount of work on the box.

13. A wrench is used to exert a force of $3.4\,\hat{\imath} + 7.8\,\hat{\jmath}$ Newtons on a wrench with a radius of $1.5\,\hat{\imath} + 0.8\,\hat{\jmath}$ meters. Calculate the torque of the wrench.